The obtained quantitative dependences confirm the qualitative analogy of the processes of chemical boiling with heat exchange upon boiling.

NOTATION

$Nu = kd/D_R$, Nusselt number; $Re = wd/\nu$, Reynolds number; $Pr = \nu/D_R$, Prandtl number; $Ar = (gd^3/\nu^2)[(\rho_R - \rho_g)/\rho_g]$, Archimedes number; $Ja_h = (c_p\Delta T/r)(\rho_l/\rho_v)$, heat Jacobi number; k, mass-transfer coefficient, m/sec; d, particle diameter, m; D, diffusion coefficient, m^2/sec; ρ, density, kg/m^2; ν, viscosity of the liquid, m^2/sec; g, acceleration of gravity, m/sec^2; c_p, heat capacity with constant pressure, J/kg·°K; r, specific heat of vaporization, J/kg; a, thermal diffusivity, m^2/sec; n, stoichiometric coefficient. Subscripts: R, reagent; g, gas; l, liquid; v, vapor.

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IMPREGNATION OF HEATED FILLER WITH VISCOPLASTIC LIQUID

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The article presents the calculation of the technological process of continuous impregnation of a heated porous layer (filler) moving in a viscoplastic liquid with the the aid of the approximate parametric method of the boundary-layer theory.

The production of many composite materials includes the impregnation of porous fillers with various liquid binders which then, as a result of polymerization or crystallization, change into the solid state. Figure 1 shows one of the models of an impregnation apparatus dealt with in [1]. In chamber I the filler is heated in a gaseous medium to the temperature T_0 (which may attain thousands of centigrades or more), and then it is drawn at constant speed u into chamber II filled with liquid binder at the temperature $T_f < T_0$. The pressure of the liquid p_f is greater than the air pressure p_0 in the pores of the filler. Under the effect of the originating pressure gradient the liquid begins to enter the filler, thereby expelling the air from its pores, and at the distances $x \geqslant L$ from the entrance into chamber II the liquid fills them completely. As a result of heat exchange the liquid is heated, its viscosity decreases, and the filtration rate increases.

From the solution of the problem of impregnation we have to determine in the first place the length L of the working section of the apparatus in dependence on the parameters of the process and the physical characteristics of the media, and a way of reducing this length while at the same time increasing the drawing speed; this is of interest in attempts to intensify the process under consideration. In the case of a filler having the form of a thin flat layer, and binder with the properties of an ordinary viscous liquid, this problem has a self-similar solution [1].
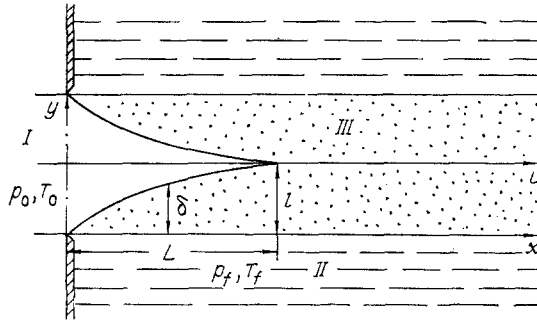
Fig. 1. Diagram of the impregnation of composite material.

The present article examines an analogous problem involving a viscoplastic binder (polymer solution, resin, etc.) which is widely used in practice. We will describe its filtration by the generalized d'Arcy law [2]

$$
v_y = \begin{cases} -\dfrac{k}{\mu}\left(\dfrac{\partial p}{\partial y} + j\right) & \text{for } \left|\dfrac{\partial p}{\partial y}\right| > j, \\ 0 & \text{for } \left|\dfrac{\partial p}{\partial y}\right| \leqslant j. \end{cases}
\tag{1}
$$

The visocity and the initial pressure gradient are decreasing functions of the temperature which we assume to be linear

$$
\mu = \mu_f(1 - m\theta), \quad j = j_f(1 - n\theta),
$$

$$
\theta = \frac{T - T_f}{T_0 - T_f}, \quad m = 1 - \frac{\mu_0}{\mu_f}, \quad n = 1 - \frac{j_0}{j_f}.
\tag{2}
$$

For a filler with small pores it may be assumed that the temperatures of the solid and the liquid phase are the same at each point of region III, and that the speed of the liquid in the direction x coincides with the speed with which the filler is drawn through. Then the equation of convective heat conduction and the expression correlating v_y with the thickness δ of the impregnated part of the layer assume the form [1]

$$
u\frac{\partial T}{\partial x} + \varkappa v_y\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2},
\tag{3}
$$

$$
v_y = v_y(x) = \varepsilon u\frac{d\delta}{dx},
$$

$$
\varkappa = \frac{\rho_f c_f}{\varepsilon\rho_f c_f + (1 - \varepsilon)\rho_s c_s}, \quad a = \frac{\lambda}{\varepsilon\rho_f c_f + (1 - \varepsilon)\rho_s c_s}.
\tag{4}
$$

Thus the problem reduces to the solution of the system (1)-(4) with the following conditions:

$$
T(x, 0) = T_f, \ T(x, \delta) = T_0, \ \delta(0) = 0,
$$

$$
p(x, 0) = p_f + p_c, \ p(x, \delta) = p_0.
\tag{5}
$$

Because of the existence of an initial pressure gradient the problem becomes non-self-similar, and for its solution we use the parametric method of the boundary-layer theory.

If we integrate (1) and (3) with respect to y from 0 to δ and take (4) and (5) into account, we obtain

$$
\delta\frac{df}{d\delta} + f - 1 + \varkappa\varepsilon = \frac{\varkappa\varepsilon}{\omega}\left[\frac{\partial\theta}{\partial\eta}(x, 1) - \frac{\partial\theta}{\partial\eta}(x, 0)\right],
\tag{6}
$$

$$
\delta\frac{d\delta}{dx} = \frac{M}{\varepsilon}\omega.
\tag{7}
$$

Here we introduce the notation:

$$
f = \int_0^1 \theta(x, \eta)\,d\eta, \quad \eta = \frac{y}{\delta}, \quad \omega = P\frac{1 - \beta\delta(1 - nf)}{1 - mf},
$$

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$$P = \frac{k\rho_f c_f \Delta p}{\mu_f \lambda}, \quad M = \frac{\lambda}{lu\rho_f c_f}, \quad \Delta p = p_f + p_c - p_0, \tag{8}$$

and the coordinates and the speed are made dimensionless by dividing then by l and u, respectively.

In accordance with (5) and (3), the function $\theta(x, \eta)$ has to satisfy the conditions

$$\theta(x, 0) = 0, \quad \theta(x, 1) = 1, \quad \omega \frac{\partial \theta}{\partial \eta}(x, 0) = \frac{\partial^2 \theta}{\partial \eta^2}(x, 0). \tag{9}$$

We choose it in the form of a polynomial of third power in respect to η:

$$\theta(x, \eta) = a_1 \eta + a_2 \eta^2 + a_3 \eta^3. \tag{10}$$

We find the coefficients a_i by using (8) and (9):

$$a_1 = \frac{6(4f - 1)}{6 + \omega}, \quad a_2 = \frac{3\omega(4f - 1)}{6 + \omega},$$

$$a_3 = \frac{4}{6 + \omega}[3(1 - 2f) + \omega(1 - 3f)].$$

Then Éq. (6), which determines the shape parameter f as a function of $\beta\delta$, assumes the form

$$\delta \frac{df}{d\delta} + f - \frac{\gamma(1 - 2f)(1 - mf)}{1 - \beta\delta(1 - nf)} - 1 + \varkappa\varepsilon = 0, \quad \gamma = 6\varkappa\varepsilon/P. \tag{11}$$

In accordance with (1), filtration is possible when the condition $\beta < 1$ is fulfilled. If we limit ourselves to the case of a small initial gradient ($\beta\delta \ll 1$), the function $f(\beta\delta)$ may be regarded as linear:

$$f = f_0 + f_1 \beta\delta. \tag{12}$$

From Eq. (11) we find:

$$f_0 = \frac{1}{4m\gamma}\{1 + (2 + m)\gamma \pm \sqrt{[1 + (2 - m)\gamma]^2 - 4m\gamma(1 - 2\varkappa\varepsilon)}\},$$

$$f_1 = \frac{\gamma(1 - 2f_0)(1 - mf_0)(1 - nf_0)}{2 + \gamma(2 + m - 4mf_0)}.$$

In front of the root sign in the expression for f_0 we have to choose the minus sign because by definition $f \leqslant 1$ for any values of the physical parameters of the medium (in particular even when $\gamma m \to 0$). In this approximation it follows from Eq. (7) that

$$\delta = \delta_0(x)\left[1 - \frac{\beta}{3}B\delta_0(x)\right], \quad \delta_0(x) = A\sqrt{x},$$

$$A = \sqrt{\frac{2k\Delta p}{\varepsilon\mu_f lu(1 - mf_0)}}, \quad B = \frac{2(1 - nf_0)[1 + \gamma(1 - mf_0)]}{2 + \gamma(2 + m - 4mf_0)}. \tag{13}$$

We can easily convince ourselves that $B > 0$.

The length of the working section of the impregnation apparatus is determined by the condition $\delta(L) = 1$ from which (passing to dimensional form) we obtain

$$L = \frac{\varepsilon\mu_f u(1 - mf_0) l^2}{2k\Delta p}\left(1 + \frac{2}{3}\beta B\right). \tag{14}$$

When $\beta = 0$, formula (14) coincides with the result obtained by Buevich and Kalinnikov [1]. Thus non-Newtonian properties of the binder lead to an increase of the relative length of the working section by $(2/3)\beta B$. The problem of a single impregnation of a heated viscoplastic liquid, taking the solubility of air in the pores of the filler into account, is dealt with in the article by Kosachevskii et al. [3].

NOTATION

u, drawing speed; v_y, filtration rate of the liquid; x, longitudinal coordinate; y, transverse coordinate; λ, thermal conductivity; k, permeability of the filler; ε, porosity

of the filler; ρ_f, density of the liquid; ρ_s, density of the filler; c_f, specific heat capacity of the liquid; c_s, specific heat capacity of the filler; p, pressure, p_c, capillary pressure; p_f, pressure of the liquid at the filler interface; p_0, air pressure in the pores of the filler; T_f, temperature at the filler interface; T_0, air temperature in the pores; μ_f, μ_0, viscosity of the liquid at the temperatures T_f, T_0, respectively; j_f, j_0, initial pressure gradients at these temperatures; $2l$, thickness of the impregnated layer; $\delta(x)$, thickness of the impregnated part of the working section of the apparatus.

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MASS TRANSFER IN A REACTIVE SPHERICAL PARTICLE

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A macrokinetic model is presented for the solid-phase transformation of a particle under arbitrary relationships between the sorption—desorption, dissolution, and diffusion rates of the initial and final products.

The problem of mass-transfer kinetics proceeding within a spherical particle during chemical or phase transformation occurs in connection with the necessity to model numerous technological processes of treating crushed and powdery materials. Theoretical information available in the literature on this subject refers primarily to situations when the macro-kinetics of the process is constrained by the rate of just some one slowest stage, absorption external or internal diffusion, etc. [1, 2]. By using the model and method in [3], a theory is constructed in this paper in which it is assumed that the rates of several stages can be of comparable magnitude. Known simplified models turn out to be particular modifications of that proposed.

We make the following assumptions, analogous to those in [3]. The system has spherical symmetry. The mixing process in the external gas or liquid phase, as well as intrinsic trans-formation ("reaction"), are realized considerably more rapidly than the processes of adsorption and desorption from the external phase and the solid solution and diffusion in the solid phase. Upon compliance with these assumptions the reaction is localized in a very narrow spherical zone separating the domains in which the reaction has still not started or has al-ready terminated, while the external phase characteristics at the particle surface are the same as far away from it.

Most important in principle is the assumption about the existence of a narrow reaction zone (a "front"). This assumption can be spoiled if the reaction rate is less than or on the order of magnitude of the rate of the mass-transfer processes. The reaction then proceeds simultaneously over the whole particle volume and the mathematical formulation of the problem is complicated substantially; at the present time only solutions obtained on the basis of approximate models, for which examples can be found in [4], are possible. Such a situation is especially characteristic for heterogeneous transformations in porous particles, although the representation of a reaction front in this case often turns out to be adequate (see, e.g., the problem of drying a porous particle in [5]).

For simplicity, we assume that there is just one substance ("reagent") that comes to the front from the external phase, and a single substance ("reaction product") that results in the